

Pseudoclassical and quantum theory of the $D=2n$ dimensional
relativistic spinning particle with anomalous “magnetic” moment
in the external Yang-Mills field *

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Abstract

The pseudoclassical hamiltonian and action of the $D = 2n$ dimensional Dirac particle with anomalous magnetic moment interacting with the external Yang-Mills field are found. The Bargmann-Michel-Telegdi equation of motion for the Pauli-Lubanski vector is deduced. The canonical quantization of $D = 2n$ dimensional Dirac spinning particle with anomalous magnetic moment in the external Yang-Mills field is carried out in the gauge which allows to describe simultaneously particles and antiparticles (massive and massless) already at the classical level. Pseudoclassical Foldy-Wouthuysen transformation is used to obtain canonical (Newton-Wigner) coordinates and in terms of these variables the theory is quantized.

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1 Introduction

As it is well known [1, 2], within the pseudoclassical approach to the theory of interaction of the point particle with external fields Grassmann variables can be used to describe not only spin degrees of freedom, but internal symmetries as well. The advantage of this is that after quantization one gets automatically finite-dimensional representations of the internal symmetry group. Following the suggestion of [1, 2] in papers [3, 4] the pseudoclassical theory of the point particle interacting with external Yang-Mills field was developed.

This paper is devoted to the construction of the pseudoclassical theory of the relativistic spinning particle with anomalous "magnetic moment" (AMM), interacting with external Yang-Mills field and its quantization. The quantization of the relativistic spinning particle with anomalous magnetic moment (AMM) interacting with external electromagnetic field was investigated in [5-8]. In paper [8] of the authors the lagrangian of the interaction of the relativistic spinning particle with AMM with the external electromagnetic field in $D = 2n$ dimensions was constructed and the Bargmann-Michel-Telegdi equation of motion for the Pauli-Lubanski vector was obtained. The quantization of the theory was carried out in terms of canonical (Newton-Wigner) coordinates, which were found using pseudoclassical Foldy-Wouthuysen transformation [9, 10]. This allowed to bypass rather cumbersome calculations of the Dirac brackets and the subsequent diagonalization of these brackets.

This paper is in the stream of the paper [8] and in essence is the generalization of it to the presence in the theory of the internal symmetries.

In sect.2 the hamiltonian and the lagrangian (action) of the $D = 2n$ -dimensional spin $\frac{1}{2}$ particle with anomalous "magnetic moment" in the external Yang-Mills field are found and the Bargmann -Michel -Telegdi equation for the Pauli -Lubansky vector is deduced. In sect.3 the Foldi -Wouthuysen transformation is used to find canonical variables, in terms of which the quantization of the theory is carried out in sect.4.

2 The Action of spin $\frac{1}{2}$ particle with anomalous magnetic moment in the external Yang-Mills field and the Bargmann- Michel- Telegdi equation

To describe the internal symmetries in the pseudoclassical theory of the interaction of the relativistic spinning particle with the external Yang-Mills field we introduce as in [1-4] Grassmann variables θ_m , $m = 1, \dots, N$. Let the transformation of the variables θ_m under the gauge transformations of the G group have the following form:

$$\delta\theta_m = iT_{mn}^a \theta_n a(x), \quad (1)$$

where T^a are generators of the given representation of the group G , satisfying the relations

$$[T^a, T^b] = if^{abc}T^c, \quad (2)$$

and which in the case of real θ_m considered here are antisymmetric; f^{abc} are the structure constants of G , $a(x)$ are parameters of the infinitesimal gauge transformations. The requirement of the invariance of the theory under the transformations (1), as is well known, necessarily leads to the introduction of the YM fields A_μ^a .

To construct the theory we postulate the constraints

$$\begin{aligned} \Phi_\mu &= \pi_\mu - \frac{i}{2}\xi_\mu, \quad \mu = 0, 1, \dots, D-1; \quad \Phi_D = \pi_{D+1} + \frac{i}{2}\xi_{D+1}, \\ \Phi_m &= \pi_m + \frac{i}{2}\theta_m, \quad m = 1, \dots, N \end{aligned} \quad (3)$$

where ξ_μ, ξ_{D+1} are Grassmann variables describing the spin degrees of freedom, $\pi_\mu, \pi_{D+1}, \pi_m$ are the momenta, canonically conjugate to ξ_μ, ξ_{D+1} and θ_m correspondingly. The constraints Φ_μ and Φ_D are the same as in the case of the particle without AMM (free or in the external field [8, 10]). The constraint Φ_m is postulated in analogy with the constraints Φ_μ, Φ_D . The Dirac brackets $\{\dots, \dots\}^*$ of the variables of the theory for the set of constraints (3) are given by the relations

$$\{x_\mu, P_\nu\}^* = g_{\mu\nu}, \quad \{\xi_\mu, \xi_\nu\}^* = ig_{\mu\nu}, \quad \{\xi_{D+1}, \xi_{D+1}\}^* = -i, \quad \{\theta_m, \theta_n\}^* = -i\delta^{mn}, \quad (4)$$

(all other brackets vanish). Here the variables x^μ are the coordinates of the particle, P_μ are the momenta conjugate to x_μ . The analogue of the fermionic constraint, which in the case of the relativistic spinning particle with AMM interacting with the external electromagnetic field [8] after quantization using brackets (4) brought to the covariant Dirac equation, in this case will be chosen in the form

$$\Phi_{D+1} = \mathcal{P}_\mu \xi^\mu - m \xi_{D+1} + i R G_{\mu\nu}^a I^a \xi^\mu \xi^\nu \xi_{D+1} \quad (5)$$

where R is a parameter of the theory, (R - analogue of the AMM of the particle in electrodynamics), $\mathcal{P}_\mu = P_\mu - g A_\mu^a I^a$, $A_\mu^a, G_{\mu\nu}^a$ are the vector-potential and the stress tensor of the YM field, I^a are the generators of the gauge transformation.

$$\delta\theta = \{\theta, a(x)I^a\} \quad (6)$$

($\{\dots, \dots\}$ denotes the Poisson bracket), which are given by the expressions (see [3, 4])

$$I^a = i\pi_m T_{mn}^a \theta_n = \frac{1}{2} \theta_m T_{mn} \theta_n \quad (7)$$

and satisfy the relations

$$\{I^a, I^b\} = f^{abc} I^c. \quad (8)$$

Note, that for T_{mn}^a , belonging to a certain representation (of the dimension N) of the L_G algebra of the gauge group G , the generators I^a represent a subalgebra of the algebra $L_{SO(N)}$ of the orthogonal group $SO(N)$, and hence the group G is a subgroup of the latter [4].

Considering now, as in [8], a theory with Dirac brackets (4) and a hamiltonian

$$H = \frac{i\chi}{2} \Phi_{D+1}, \quad (9)$$

where χ is the Grassmann odd Lagrange multiplier to Φ_{D+1} , we find from the consistency condition a new constraint Φ_{D+3}

$$\begin{aligned} \{\Phi_{D+1}, \Phi_{D+1}\}^* &= i\Phi_{D+3} = i(\mathcal{P}_\mu \mathcal{P}^\mu - ig M G_{\mu\nu}^a I^a \xi^\mu \xi^\nu + \\ &+ 4i R G_{\mu\nu}^a I^a \mathcal{P}^\mu \xi^\nu \xi_{D+1} + R^2 (G_{\mu\nu}^a I^a \xi^\mu \xi^\nu)^2 - m^2) \approx 0, \end{aligned} \quad (10)$$

where $gM = (g - 2Rm)$, M is the total "magnetic" moment of the particle. To deduce (10) we had made use of the relation

$$\{\mathcal{P}_\mu, \mathcal{P}_\nu\}^* = gG_{\mu\nu} \equiv gG_{\mu\nu}^a I^a, \quad (11)$$

which can be easily checked by direct calculation, and also of the identity

$$\begin{aligned} \{G_{\mu\nu}, \mathcal{P}_\lambda\}^* \xi^\mu \xi^\nu \xi^\lambda &= \nabla_\lambda^{ab} G_{\mu\nu}^b I^a \xi^\mu \xi^\nu \xi^\lambda \equiv 0, \\ \nabla_\lambda^{ab} &= \delta^{ab} \partial_\lambda + g f^{abc} A_\lambda^c \end{aligned} \quad (12)$$

which can be derived using (11) and the Jacoby identity

$$\{\{\mathcal{P}_\mu \xi^\mu, \mathcal{P}_\nu \xi^\nu\}^*, \mathcal{P}_\lambda \xi^\lambda\}^* \equiv 0. \quad (13)$$

Now taking into account the constraint Φ_{D+3} we can construct the extended hamiltonian

$$H_{\text{ext}} = \frac{i\chi}{2} \Phi_{D+1} + \frac{e}{2} \Phi_{D+3}, \quad (14)$$

where e is Grassmann even Lagrange multiplier to the constraint Φ_{D+3} . The consistency equations imply no new constraints since due to Jacoby identity

$$\{\Phi_{D+3}, \Phi_{D+1}\}^* = \{\{\Phi_{D+1}, \Phi_{D+1}\}^*, \Phi_{D+1}\}^* = 0. \quad (15)$$

Hence the dynamics of the theory is described by the hamiltonian (14)

The action (lagrangian) of the relativistic spinning particle with AMM in the external YM field can be found by Legendre transformation using (14) and the constraints (3). In this way we find that

$$\begin{aligned} S = & \frac{1}{2} \int d\tau \left[\frac{(\dot{x}^\mu)^2}{e} + em^2 - i \left(\xi_\mu \dot{\xi}^\mu - \xi_{D+1} \dot{\xi}_{D+1} \right) + i \theta_m \dot{\theta}_m + \right. \\ & + 2g \dot{x}^\mu A_\mu^a I^a + igMeG_{\mu\nu}^a I^a \xi^\mu \xi^\nu - 4iRG_{\mu\nu}^a I^a \dot{x}^\mu \xi^\nu \xi_{D+1} - \\ & \left. - i\chi \left(\frac{\xi_\mu \dot{x}^\mu}{e} - m\xi_{D+1} - iRG_{\mu\nu}^a I^a \xi^\mu \xi^\nu \xi_{D+1} \right) - eR^2 (G_{\mu\nu}^a I^a \xi^\mu \xi^\nu)^2 \right], \end{aligned} \quad (16)$$

where the overdot denotes the differentiation with respect to τ along the trajectory of the particle.

Following [8] we write down the complete set of the constraints of the theory

$$\Phi_\mu = \pi_\mu - \frac{i}{2}\xi_\mu, \quad \Phi_{D+1} = \pi_{D+1} + \frac{i}{2}\xi_{D+1}, \quad \Phi_m = \pi_m + \frac{i}{2}\theta_m, \quad (17)$$

$$\begin{aligned} \Phi_{D+3} = & \mathcal{P}_\mu \mathcal{P}^\mu - igMG_{\mu\nu}^a I^a \xi^\mu \xi^\nu + \\ & + 4iRG_{\mu\nu}^a I^a \mathcal{P}^\mu \xi^\nu \xi_{D+1} + R^2 (G_{\mu\nu}^a I^a \xi^\mu \xi^\nu)^2 - m^2 \end{aligned} \quad (18)$$

$$\Phi_{D+4} = x'_o, \quad (19)$$

$$\Phi_{D+5} = \pi_e, \quad \Phi_{D+6} = e - \frac{\kappa}{\mathcal{P}_o}, \quad (20)$$

$$\Phi_{D+1} = \mathcal{P}_\mu \xi^\mu - m\xi_{D+1} + iRG_{\mu\nu}^a I^a \xi^\mu \xi^\nu \xi_{D+1}, \quad \Phi_{D+2} = a\xi_o + b\xi_{D+1}. \quad (21)$$

Here π_e is the canonical momentum, conjugate to the e ; $\kappa = \pm 1$, $\kappa = +1$ corresponds to the presence of the particle in the theory, while $\kappa = -1$ to that of antiparticle; a and b are parameters of the theory, $a^2 + b^2 \neq 0$. When $a \neq 0$, the theory has the massless limit ($m \rightarrow 0$) [11].

The system (17-21) includes the constraints, which follow from the action (16) using Dirac procedure of the evaluation of new constraints, as well as additional constraints, introduced into the theory for a complete fixation of the gauges, so that now all the constraints are second class. The additional constraints are $\Phi_{D+4}, \Phi_{D+6}, \Phi_{D+2}$. Note [11], that $x'_o = x_o - \kappa\tau$ and the constraint $\Phi_{D+4} = x_o - \kappa\tau$ transforms into (19) after a canonical transformation from the variables x^μ, p_μ to variables x'^μ, p'_μ , defined by the relations

$$x'_o = x_o - \kappa\tau, \quad x'^i = x^i, \quad p'_\mu = p_\mu \quad (22)$$

Note also, that we omitted in the complete set of constraints ((17)-(21)) the pair $\chi \approx 0$, $\pi_\chi \approx 0$ (π_χ is the momentum, canonically conjugate to the variable χ), since the Dirac brackets for this subset of constraints are equal to the Poisson brackets for the variables of the theory.

To end this section, we will briefly comment on the Bargmann-Michel-Telegdi equation for Pauli-Lubanski vector W_μ , which in $D = 2n$ dimensions is defined as [10]

$$\begin{aligned} W_\mu = & \frac{(-i)^{\frac{D-2}{2}}}{(D-2)!} \varepsilon_{\mu\nu\lambda_2\lambda_3\dots\lambda_{D-1}} \mathcal{P}^\nu \xi^{\lambda_2} \xi^{\lambda_3} \dots \xi^{\lambda_{D-1}}, \\ & \mathcal{P}^\nu = p^\nu - gA^{\nu,a} I^a. \end{aligned} \quad (23)$$

Going through the same steps as in [8] we obtain the desired equation

$$\dot{W}_\mu = g \frac{M}{m} G_{\mu\nu}^a I^a W^\nu + 2R u_\mu G_{\nu\lambda}^a I^a W^\nu u_\lambda + O(\xi^{(D)}). \quad (24)$$

where u_μ is the D -velocity: $u_\mu = \dot{x}_\mu = \{x_\mu, H\}^*$, $H = H_{\text{ext}}|_{\chi=0, e=1/m}$.

3 Foldy-Wouthuysen Transformation and the canonical variables

As it was mentioned in [10], it is convenient to quantize the theory not in terms of the initial variables, but in terms of canonical variables, because the Dirac brackets of the independent original variables in the presence of the external fields are very complicated and the operator realization of the theory in terms of the original variables seems improbable. The canonical variables can be found either by diagonalization of the Dirac brackets, as it was done in [12], or by using pseudoclassical canonical Foldy - Wouthuysen transformation, as it was done in [8, 10]. We take a generator of the infinitesimal canonical transformations [9] in the form

$$S = -2i (\mathcal{P}_j \xi_j) \xi_{D+1} \theta(\gamma), \quad (25)$$

Here $\gamma = i\{(\mathcal{P}_i \xi_i), (\mathcal{P}_j \xi_j)\}^* = \mathcal{P}_i^2 + ig G_{ij}^a I^a \xi_i \xi_j$, and $\theta(\gamma)$ satisfies the relation $tg(2\theta\sqrt{\gamma}) = \frac{\sqrt{\gamma}}{\tilde{m}}$, where $\tilde{m} = m - iR G_{ij}^a I^a \xi_i \xi_j$. The result of the finite canonical transformation of any dynamical quantity A is given by the expression [13]

$$\tilde{A} = e^{\widetilde{S}} A = A + \{A, S\}^* + \frac{1}{2!} \{\{A, S\}^*, S\}^* + \dots, \quad (26)$$

Repeating now the reasoning of [10] one can prove the following relation

$$\begin{aligned} \{\tilde{A}, \tilde{B}\}_{D(\Phi)} &= \{A', B'\}_{D(\Phi)} = \\ &= \{A, B\}^* + i\{\{A, B\}^*, (\mathcal{P}_i \xi_i)\}^* (\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})}, \end{aligned} \quad (27)$$

where

$$A' \equiv \tilde{A}|_{\Phi=0} = A + i\{A, (\mathcal{P}_i \xi_i)\}^* (\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})}, \quad (28)$$

$\tilde{\beta} = a\tilde{m} - b\kappa\omega$, $\omega = \sqrt{\mathcal{P}_i^2 + \tilde{m}^2 + igG_{ij}^a I^a \xi_i \xi_j}$, and $\{\cdot, \cdot\}_{D(\Phi)}$ denotes Dirac bracket for the complete set of constraints (17-21). The first equality in (27) is the reflection of the property of the Dirac brackets, according to which the Dirac brackets of the constraints with any dynamical variable vanish. The relations (27), (28) are crucial for finding the canonical variables. If now we take for A, B the variables $x_i, \mathcal{P}_j, \xi_k, \theta_m$, then on account of (28) we find

$$x'_i = x_i - i\xi_i(\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})} \equiv q_i, \quad (29)$$

$$\mathcal{P}'_i = \mathcal{P}_i + igG_{ij}^a I^a \xi_j(\mathcal{P}_k \xi_k) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})} \equiv \pi_i, \quad (30)$$

$$\xi'_i = \xi_i + \mathcal{P}_i(\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})} \equiv \psi_i, \quad (31)$$

$$\theta'_m = \theta_m - g(A_i^a \xi_i) T_{ml}^a \theta_l(\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})} \equiv \varphi_m. \quad (32)$$

Applying now the formula (27) to variables $q_i, \pi_j, \psi_i, \varphi_m$ we find the relations

$$\{q_i, \pi_j\}_{D(\Phi)} = \{x_i, \mathcal{P}_j\}^* = -\delta_{ij}, \quad (33)$$

$$\{\pi_i, \pi_j\}_{D(\Phi)} = gG_{ij}^a(x) I^a + ig \left(\nabla_k^{ab} G_{ij}^b(x) \right) I^a \xi_k(\mathcal{P}_m \xi_m) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})} = gG_{ij}^a(q) I_\varphi^a, \quad (34)$$

$$\{\psi_i, \psi_j\}_{D(\Phi)} = \{\xi_i, \xi_j\}^* = -i\delta_{ij}, \quad (35)$$

$$\{\varphi_m, \varphi_n\}_{D(\Phi)} = \{\theta_m, \theta_n\}^* = -i\delta_{mn}, \quad (36)$$

(all other brackets vanish). From these relations it is clear, that the variables $q_i, \pi_j, \psi_k, \varphi_m$, which are just the variables $\tilde{x}_i, \tilde{\mathcal{P}}_j, \tilde{\xi}_k, \tilde{\theta}_m$ on the constraint surface, are canonical variables (they have canonical Dirac brackets).

For the variable $I'^a \equiv I_\varphi^a = \frac{1}{2}\varphi T^a \varphi$ one can find from (28) taking into account (8)

$$I'^a = I^a - igf^{abc}(A_i^b \xi_i) I^c(\mathcal{P}_j \xi_j) \frac{(b\kappa + a)}{\tilde{\beta}(\omega + \tilde{m})}, \quad (37)$$

then from (27), with the account of (8) and (37) one can deduce, that

$$\{I_\varphi^a, I_\varphi^b\}_{D(\Phi)} = f^{abc} I_\varphi^c. \quad (38)$$

From formulae (29-32) one can find the expressions of initial variables $x_i, \mathcal{P}_j = p_j - gA_j^a I^a, \xi_k, \theta_m$ in terms of new variables $q_i, \pi_j = \Pi_j - gA_j^a(q)I^a, \psi_k, \varphi_m$ (Π_i is the momentum, canonically conjugate to q_i):

$$x_i = q_i - i\psi_i(\pi_j\psi_j)\frac{(a\kappa + b)}{\tilde{\alpha}(\Omega + m_\psi)}, \quad (39)$$

$$\mathcal{P}_i = \pi_i + igG_{ij}^a(q)I_\varphi^a\psi_j(\pi_k\psi_k)\frac{(a\kappa + b)}{\tilde{\alpha}(\Omega + m_\psi)}, \quad (40)$$

$$\xi_i = \psi_i + \pi_i(\pi_j\psi_j)\frac{(a\kappa + b)}{\tilde{\alpha}(\Omega + m_\psi)}, \quad (41)$$

$$\theta_m = \varphi_m - gT_{ml}^a\varphi_l(A_i^a(q)\psi_i)(\pi_j\psi_j)\frac{(a\kappa + b)}{\tilde{\alpha}(\Omega + m_\psi)}, \quad (42)$$

where $\tilde{\alpha} = -a\kappa\Omega + bm_\psi, m_\psi = m - iRG_{ij}^a(q)I_\varphi^a\psi_i\psi_j, \Omega = \sqrt{\pi_i^2 + m_\psi^2 + igG_{ij}^a(q)I_\varphi^a\psi_i\psi_j}$.

The formula inverse to (37) has the form

$$I^a = I_\varphi^a - igf^{abc}(A_i^b(q)\psi_i)I_\varphi^c(\pi_j\psi_j)\frac{(a\kappa + b)}{\tilde{\alpha}(\Omega + m_\psi)}, \quad (43)$$

4 Quantization

The transition to the quantum theory is realized by the replacement of the generalized coordinates and momenta by operators, for which the commutators are defined by the rule $[\cdot, \cdot] = i\hbar\{\cdot, \cdot\}_D$. Introducing operators $\hat{q}_i, \hat{\Pi}_j, \hat{\psi}_k, \hat{\varphi}_m$ corresponding to canonical variables $q_i, \Pi_j, \psi_k, \varphi_m$ we have

$$[\hat{q}^i, \hat{\Pi}_j]_- = \hbar\delta_j^i, \quad [\hat{\psi}^i, \hat{\psi}^j]_+ = \hbar\delta^{ij}, \quad [\hat{\varphi}_m, \hat{\varphi}_n]_+ = \hbar\delta_{mn} \quad (44)$$

(all other brackets vanish). The last two of these relations generate a Clifford algebras in $D - 1$ and N dimensional spaces correspondingly. The finite dimensional matrix representations of these algebra are given by the $2^{(D-2)/2} \otimes 2^{(D-2)/2}$ matrices $\sigma_i, i = 1, \dots, D - 1$, and by $2^{[N/2]} \otimes 2^{[N/2]}$ matrices $\Sigma_m, m = 1, \dots, N$.

$$\hat{\psi}_i = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} \sigma_i, \quad \hat{\varphi}_m = \left(\frac{\hbar}{2}\right)^{\frac{1}{2}} \Sigma_m, \quad (45)$$

The isospin variable operator I_φ^a is then given by

$$\hat{I}_\varphi^a = \frac{\hbar}{4} \Sigma_m T_{mn}^a \Sigma_n \quad (46)$$

and belongs to the spinor representation of the algebra $L_{SO(N)}$. Now, since , as it was mentioned above, the group G is a subgroup of $SO(N)$, to find the isospin content of the quantum states one must decompose this spinor representation over the irreducible representations of G [4].

For the sake of completeness we will give the expression for the quantum Hamiltonian of the theory

$$\begin{aligned} \hat{H}_{\text{phys}} \overset{\leftrightarrow}{=} & \Omega - g\kappa A_0^a I^a - ig\kappa \frac{G_{0k}^a I^a \psi_k (\pi_j \psi_j)}{\Omega(\Omega + \tilde{m}_\psi)} + \\ & + \frac{2iR\kappa}{\Omega} \left(G_{0k}^a I^a \psi_k + \frac{\kappa G_{ik}^a I^a \pi_i \psi_k}{\Omega + \tilde{m}_\psi} \right) (\pi_j \psi_j). \end{aligned} \quad (47)$$

where $\overset{\leftrightarrow}{=}$ mens the Weyl correspondence between the symbols and their operators. *Acknowledgment*

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